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FILM BOILING HEAT TRANSFER FROM A HORIZONTAL

SURFACE AS AN OPTIMAL BOUNDARY VALUE PROCESS

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ABSTRACT

A theoretical analysis of pool film boiling based on a cellular model is presented. The time-averaged cell configuration is postulated to adjust itself to maximize the rate of heat transfer. Theoretical expressions for the heat-transfer coefficient, average vapor dome (bubble) radius, and optimum cell diameter (wavelength) were in close agreement with experiments. As a generalization of this work and others, a tentative nonequilibrium thermodynamic hypothesis is proposed that nature maximizes the entropy production in all situations characterized by macroscopic chaos.

AUSZUG

Eine theoretische Analyse einer siedenden Oberfläche auf der Grundlage eines zellenartigen Modells wird gegeben. Die zeitgemittelte Zellenkonfiguration stellt sich voraussetzungsgemäss so ein dass sie die Geschwindigkeit der Wärmeübertragung maximiert. Ein theoretischer Ausdruck für die Wärmeleitzahl, den Dampfdomradius (Blase) und den optimalen Zellendurchmesser stimmte sehr nahe mit Versuchen überein. Als eine Verallgemeinerung dieser Arbeit und anderer wird eine vorläufige ausgleichslose thermodynamische Hypothese vorgebracht, nämlich dass die Natur die Erzeugung der Entropie in allen durch makroskopisches Chaos charakterisierten Umständen maximiert.

АННОТАЦИЯ

Дан теоретический анализ кипения пленки на горизонтальной поверхности в предположении целлюлярной модели. Предполагается что усредненная во времени конфигурация ячейки самонастраивается на максимальную скорость теплопередачи. Теоретические выражения для коэффициента теплопередачи, среднего радиуса пузыря и оптимального диаметра (длины волны) ячейки находятся в хорошем согласовании с экспериментальными данными. Как обобщение этой и других работ условно предлагается гипотеза термодинамической неуравновешенности при которой природа сама минимизирует создание энтропии во всех случаях макроскопического хаоса.

INTRODUCTION

Film-boiling heat transfer in pool systems is a subject of considerable interest, especially in the field of cryogenics, since relatively low wall temperatures will induce film boiling. A layer of dense fluid supported by a lighter one, for film boiling of liquid above vapor, is known to be an inherently unstable configuration in a gravitational field. At certain locations on the vapor-liquid interface, vapor will break through and escape under the influence of gravity into the bulk liquid. Photographs of film boiling show that these escape points are dome shaped and look like large bubbles and the surface is covered by a cell-type structure.

Figure 1 illustrates the two-dimensional model used in the analysis. The model consists of a thin film of vapor between the wall and the supported liquid with the vapor domes distributed symmetrically over the surface of the plate. This model is similar to that used by Berenson [1];[1] however, the analysis used in this paper is different from that used by Berenson in that no recourse is made to experimental results or to one-dimensional small amplitude stability theory.

The present paper takes the point of view that heat transfer is the dominating or driving force for both the flow and the instabilities; that is, the time-averaged configuration of the liquid-vapor interface is determined by thermal energy considerations.

BASIC MODEL AND EQUATIONS

The major portion of the heat transport to a unit cell (see Fig. 1) is assumed to occur across the thin vapor film. The vapor domes are considered so thick that essentially no heat is conducted into them. Physically, their function is to act as hydrodynamic sinks into which the generated vapor is dumped. The assumed symmetrical distribution of these sinks implies that the velocity field in the thin film is radially symmetric. The model proposed assumes that there is some time-averaged or ensemble-averaged configuration of the system, where all velocity, pressure, and temperature fields are at steady state and that this statistically idealized configuration represents the average behavior of the actual system.

The heat-transfer coefficient can be obtained by solving the momentum and thermal energy equations for flow and heat transport in the thin annular vapor film belonging to a single cell. In order to solve the momentum equations the inertia terms in the Navier-Stokes equation are considered negligible. This assumption can be justified on the grounds that the small thickness of the film prevents the buildup of sizeable velocities. Additional support is given to this assumption by noting in [2 and 3], which consider the Leidenfrost film boiling of drops, that the difference between the solutions obtained by retaining the inertia terms and by dropping them is shown to be negligible. In addition, the physical properties are assumed constant (but evaluated at the film

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[1]Numbers in brackets denote references.

temperature); radiation and radial temperature gradients in the energy relations are assumed negligible.

For the assumptions noted, the governing differential equations in cylindrical coordinates are as follows:

Momentum:

$$0 = -g_c \frac{\partial P}{\partial r} + \mu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) \quad (1)$$

$$0 = -g_c \frac{\partial P}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) \quad (2)$$

Continuity:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \quad (3)$$

Energy:

$$w \frac{\partial T}{\partial z} = \alpha \frac{\partial^2 T}{\partial z^2} \quad (4)$$

The attendant boundary conditions are as follows:

(a) At the wall, $z = 0$. The velocity of the vapor is zero:

$$u(r,0) = w(r,0) = 0 \quad (5)$$

The temperature of the wall is at a specified constant value:

$$T(r,0) = T_w \quad (6)$$

(b) At the liquid-vapor interface,

$$z = \delta$$

The radial velocity vanishes:

$$u(r,\delta) = 0 \quad (7)$$

The normal velocity has some constant value dependent on both momentum and energy considerations, and the surface is at the saturation temperature:

$$w(r,\delta) = w_\delta = \text{constant}$$
$$T(r,\delta) = T_s \quad (8)$$

Since the bulk liquid is assumed to be at the saturation temperature T_s, no heat is conducted into the bulk liquid. The momentum and energy equations are coupled at the interface by a balance of latent heat release and heat conduction through the vapor film to the evaporating surface:

$$k\left(\frac{\partial T}{\partial z}\right)_{z=\delta} = \rho \lambda w_\delta \quad (9)$$

A static force balance requires that the average pressure of the vapor over the annular interface be equal to the system pressure P_s at the flat portion of the liquid-vapor interface, as shown in Figure 1:

$$2\pi \int_{R_1}^{R_0} P(r,\delta) r \, dr = \pi P_s (R_0^2 - R_1^2) \quad (10)$$

One further assumption is made about the pressure field in the next section to make the problem determinate.

(c) Also, from radial symmetry,

$$u(R_0, z) = 0 \quad (11)$$

METHOD OF SOLUTION

A solution flow chart of the entire analysis is presented in Figure 2. This chart contains the overall logic of the analysis and depicts how each subproblem ties in with the final results. Reference to this chart after reading each subproblem section will aid in understanding the analysis.

Stream Function

The two-dimensional axisymmetric equations of motion with inertia terms neglected, often referred to as the "creeping motion" equations, are handled most easily by use of the stream function ψ defined by

$$u = \frac{1}{r} \frac{\partial \psi}{\partial z} \quad (12)$$

$$w = -\frac{1}{r} \frac{\partial \psi}{\partial r} \quad (13)$$

The continuity equation is identically satisfied by these definitions. The momentum equations become

$$E^4 \psi = 0 \quad (14)$$

where E^2 is the operator

$$E^2 = \frac{\partial^2}{\partial r^2} - \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \quad (15)$$

Solving Equation (14) for this problem, by separation of variables, results in

$$\psi = \left(\frac{C_1 r^2}{2} + C_2 \right) \left(A_0 + A_1 z + A_2 z^2 + A_3 z^3 \right) \quad (16)$$

Substituting ψ into Equations (12) and (13) and applying the boundary conditions (Eqs. (5), (7), and (8)) result in the following expressions for the velocity components:

$$w = \beta^2 \left(z^3 - \frac{3}{2} \delta z^2 \right) \quad (17)$$

$$u = -\frac{3\beta^2}{2} \left(r - \frac{R_0^2}{r} \right) (z^2 - \delta z) \quad (18)$$

where β is a real number and, at this point, completely arbitrary.

Pressure Function

The velocity expressions (Eqs. (17) and (18)) can be substituted into the Navier-Stokes equations to obtain

$$\frac{\partial P}{\partial z} = \frac{3\mu}{g_c} \beta^2 (2z - \delta) \quad (19)$$

$$\frac{\partial P}{\partial r} = - \frac{3\mu}{g_c} \beta^2 \left(r - \frac{R_0^2}{r} \right) \qquad (20)$$

These expressions can be integrated, subject to the boundary condition

$$P(R_1,\delta) = P_s - R_1(\rho_l - \rho) \frac{g}{g_c} + \frac{2\sigma}{R_1} \qquad (21)$$

to yield as a final expression for the pressure

$$P = P_s + \frac{3\mu}{g_c} \beta^2 \left(\frac{R_1^2 - r^2}{2} + R_0^2 \ln \frac{r}{R_1} \right) - R_1(\rho_l - \rho) \frac{g}{g_c}$$
$$+ \frac{2\sigma}{R_1} + \frac{3\mu\beta^2}{g_c} (z^2 - \delta z) \qquad (22)$$

A derivation of Equation (21) appears in [1]. It suffices to say that the pressure in the dome is slightly less than the pressure P_s at the liquid-vapor interface. This difference is due to the difference in static head corrected for surface tension and curvature.

In order to determine the constant β^2, the pressure function (Eq. (22)) is inserted into the force balance (Eq. (9)), which when integrated gives

$$\beta^2 = - \frac{g_c}{3\mu R_0^4} \frac{R_0^3(\rho_l - \rho) \frac{g}{g_c} \frac{f(1-f^2)}{2} - \frac{\sigma R_0(1 - f^2)}{f}}{\frac{1 - f^4}{8} + \frac{(1 - f^2)^2}{4} + \frac{\ln f}{2}} \qquad (23)$$

where $f = R_1/R_0$. A criterion for evaluating R_0 and R_1 will be postulated in the optimum configuration section.

One of the main results of this analysis is that there is a unique relation between the rate of evaporation necessary to support the liquid and the thickness of the gap. This relation is obtained by substituting $z = \delta$ into Equation (17):

$$w_\delta = - \frac{1}{2} \beta^2 \delta^3 \qquad (24)$$

Energy Equation

Integrating Equation (4) subject to the boundary conditions of fixed temperatures T_w and T_s at both the wall and the interface, respectively, yields the general solution (as differentiation will verify)

$$\frac{T(z) - T_w}{T_s - T_w} = \frac{\int_0^z dx \exp \left[\int_0^x \frac{w(y)}{\alpha} dy \right]}{\int_0^\delta dx \exp \left[\int_0^x \frac{w(y)}{\alpha} dy \right]} \qquad (25)$$

where x and y are dummy variables.

The interface energy balance is obtained by combining Equations (9), (24), and the first derivative of Equation (25) evaluated at $z = \delta$:

$$- \frac{\rho\lambda\beta^2\delta^3}{2} = +k \frac{(T_s - T_w)\exp \left[\int_0^\delta \frac{w(y)}{\alpha} dy \right]}{\int_0^\delta dx \exp \left[\int_0^x \frac{w(y)}{\alpha} dy \right]} \qquad (26)$$

Substituting for w from Equation (17), expanding the exponential function, integrating each term, neglecting terms of order δ^8 and higher, and solving for the gap thickness gives

$$\delta = \left[\frac{2k(T_w - T_s)}{\rho\beta^2 \left(\lambda + \frac{7}{20} C_p\Delta T \right)} \right]^{1/4} \qquad (27)$$

Heat-Transfer Coefficient

Once the gap thickness is known, it is a relatively simple matter to compute the heat-transfer coefficient, which is defined by

$$h_{ann} = \frac{-k \left. \frac{dT}{dz} \right|_{z=0}}{(T_w - T_s)} \qquad (28)$$

where h_{ann} is the local heat-transfer coefficient based on the annular area surrounding the dome. The heat-transfer coefficient of interest, however, is based on the total area of the plate. If heat transfer in the dome itself is neglected, then h_{cell}, the heat-transfer coefficient based on the total area of a cell, is related to h_{ann} by

$$h_{cell} = h_{ann} \frac{A_{ann}}{A_{cell}} = h_{ann}(1 - f^2) \qquad (29)$$

Employing the first derivative of Equation (25) at $z = 0$, Equations (27), (28), and (29), and neglecting terms of δ^8 and higher results in the following heat-transfer coefficient:

$$h_{cell} = \left[\frac{k^3 \lambda^* \beta^2 \rho}{2(T_w - T_s)} \right]^{1/4} (1 - f^2) \qquad (30)$$

where

$$\lambda^* = \lambda \left(1 + \frac{19}{20} \frac{C_p\Delta T}{\lambda} \right) \qquad (31)$$

This expression for the modified latent heat of vaporization was found by repeated application of the binomial theorem, while assuming that $C_p\Delta T/\lambda$ was small.

OPTIMUM CONFIGURATION OF LIQUID-VAPOR INTERFACE

It will be shown in this section that there is an upper limit to the rate of heat transfer to a unit cell; that is, there exist "optimum" values (from the point of view of maximum heat transport) of the radii R_0 and R_1, because very large cells and very small cells are incapable of maximizing the heat transport. The heat transport to the entire system will be maximized if

$$\frac{\partial h_{cell}}{\partial R_1}\bigg|_{R_0} \simeq \frac{\partial h_{cell}}{R_0 \partial f}\bigg|_{R_0} = \frac{\partial h_{cell}}{\partial f}\bigg|_{R_0} = 0 \quad (32)$$

and

$$\frac{\partial h_{cell}(R_0,R_1)}{\partial R_0}\bigg|_{R_1} \simeq \frac{\partial h_{cell}(R_0,f)}{\partial R_0}\bigg|_{f} = \frac{\partial \beta^2}{\partial R_0}\bigg|_{f} = 0 \quad (33)$$

(These expressions were checked to insure that they did not lead to a minimum or a saddle point.) Consider Equation (33) first. Taking the derivative of β^2, given by Equation (23), with respect to R_0, setting this equal to zero, and solving for R_0 yield

$$R_0 = \left[6 \frac{\sigma g_c}{g(\rho_l - \rho)} \frac{1}{f^2}\right]^{1/2} \quad (34)$$

By substituting the value of R_0 (Eq. (34)) into the expression for β^2 (Eq. (23)) the heat-transfer coefficient given by Equation (30) becomes

$$h_{cell} = \left[\frac{k^3 \lambda^* \rho g(\rho_l - \rho)}{\mu(T_w - T_s)l}\right]^{1/4}$$

$$\times \left[\frac{-1}{3(6)^{3/2}} \frac{f^2(1 - f^2)}{\frac{(1 - f^2)^2}{4} + \frac{1 - f^4}{8} + \frac{\ln f}{2}}\right]^{1/4} (1 - f^2) \quad (35)$$

where

$$l = \left[\frac{g_c \sigma}{g(\rho_l - \rho)}\right]^{1/2} \quad (36)$$

The derivative of h_{cell} with respect to f, which was indicated by Equation (32), gives a complicated transcendental equation for the optimum value of f. It is much simpler to obtain the maximum value of h_{cell} numerically. The optimum value was found numerically to be

$$f_{opt} = 0.673 \quad (37)$$

Thus, if Equation (37) is substituted into Equation (35), the maximum value of the heat-transfer coefficient consistent with the proposed model is

$$h_{cell} = 0.381 \left[\frac{k^3 \lambda^* \rho g(\rho_l - \rho)}{\mu(T_w - T_s)l}\right]^{1/4} \quad (38)$$

The optimum cell radius from Equation (34) is

$$R_0 = 3.64\, l \quad (39)$$

and the optimum dome radius from Equation (37) is

$$R_1 = 2.45\, l \quad (40)$$

while the value of β^2 is

$$\beta^2 = 0.4716 \frac{[(\rho_l - \rho)g]^{3/2}}{\mu(\sigma g_c)^{1/2}} \quad (41)$$

In the top view of the cellular model (Fig. 1(a)), small curved triangular regions exist on the plate that were not compensated for in the analysis. These curved triangular patches must be taken into account in order to evaluate the total heat-transfer rate. Since there are six triangular patches in contact with a single cell and three cells bordering each patch, the additional area attributed to a single cell is twice the area of a triangular patch. Drawing an equilateral triangle through the centers of three adjacent cells and evaluating the areas of the enclosed circular segments readily shows that the area of the triangular patch $A_\triangle$ is given by

$$A_\triangle = R_0^2\left(\sqrt{3} - \frac{\pi}{2}\right) \quad (42)$$

The assumption is made that the local heat-transfer coefficient in the triangular patch region is the same as the heat-transfer coefficient in the annular region surrounding a vapor dome. This assumption is based on the fact that the film thickness under the triangular patch will be equal to the film thickness under the annular area. Therefore,

$$Q_{tot} = h_{ann}(A_{ann} + 2A_\triangle)\Delta T = h_{tot}(A_{cell} + 2A_\triangle)\Delta T \quad (43)$$

Solving Equation (43) for h_{tot} and combining the result with Equations (29) and (42) yield

$$h_{tot} = \left(\frac{A_{ann} + 2A_\triangle}{A_{cell} + 2A_\triangle}\right)\left(\frac{A_{cell}}{A_{ann}}\right) h_{cell} \quad (44)$$

The area ratios are evaluated from Equations (37) and (42); consequently, the heat-transfer coefficient (Eq. (38)) corrected for the triangular patches becomes

$$h_{tot} = 0.410 \left[\frac{k^3 \lambda^* \rho g(\rho_l - \rho)}{\mu(T_w - T_s)l}\right]^{1/4} \quad (45)$$

COMPARISON OF THEORETICAL AND EXPERIMENTAL RESULTS

Equation (45) is identical in form to an experimentally confirmed expression proposed by Berenson [1]. The only differences between the two results are the value of the prefactor constant and the definition of the modified latent heat of vaporization, but the lower value of the prefactor constant in the present results is compensated for by a higher modified latent heat, as shown in the following table:

Parameter	Berenson [1]	Present results
Prefactor constant	0.425	0.410
Modified latent heat of vaporization	$\lambda\left(1 + 0.5\frac{c_p \Delta T}{\lambda}\right)$	$\lambda\left(1 + \frac{19}{20}\frac{c_p \Delta T}{\lambda}\right)$

In Berenson's work, an empirical expression was used for the diameter of the dome. From a series of high-speed motion pictures of film

boiling of n-pentane and carbon tetrachloride, the diameters of the vapor domes that had just departed or were just about to depart were measured at various heat fluxes. This characteristic dome diameter was correlated by the equation

$$D_B = 4.7 \left[\frac{g_c \sigma}{g(\rho_l - \rho)} \right]^{1/2} \pm 10 \text{ percent} \quad (46)$$

This value is very close to the optimum vapor dome diameter given by Equation (40); namely,

$$D_1 = 4.90 \left[\frac{g_c \sigma}{g(\rho_l - \rho)} \right]^{1/2} \quad (47)$$

DISCUSSION OF RESULTS

The formulation of the film boiling process in terms of a maximization principle used herein is a specific instance of what is called a "generalized optimal boundary value process." Krzywoblocki in a recent work [4] has formulated a very general statement of this kind of problem. In other words, all boundary value problems can be looked upon as control problems. The boundary conditions are the controls. The optimal boundary value problem is to select from all admissible boundary conditions a set for which some functional assumes an extreme (maximum or minimum) value. It is to be emphasized that the shape of the boundary is a boundary condition. Apparently, certain kinds of physical situations are naturally optimal boundary value processes. Krzywoblocki [4] gives some examples of this natural class of optimal or extremum processes, one of them being the dynamics of a liquid enclosed in a partly filled tank under conditions of zero gravity. Here, the unknown boundary conditions (distribution of vapor and liquid) can be determined statistically by a variational principle.

In the film boiling problem, since the vapor-liquid boundary is flexible, it is conceptually possible for the vapor liquid boundary to adjust itself to maximize heat transfer. In terms of the foregoing nomenclature, a generalized postulate on film boiling is proposed: Film boiling on a horizontal surface is an optimal boundary value process where the functional to be maximized is the total rate of heat transfer. Of all the possible configurations of the vapor-liquid interface, that one which maximizes the energy transport is realized.

The final justification for the postulate is that results employing it are consistent with experimental data, as shown in the preceding section of this report; however, the applicability of a maximum principle may even transcend this particular application to film boiling. In [5] for example, Malkus advanced a novel theory on the nature of turbulent free convection. He proposed that in thermal turbulence the flow adjusts itself in such a way as to transfer the maximum amount of heat compatible with the boundary conditions. Without introducing experimental constants, a reasonably accurate prediction of the mean velocity distribution for the turbulent flow was achieved. Other papers by Malkus [6 and 7] which treat finite-amplitude cellular convection and a theory of shear turbulence, respectively, also employ a maximization postulate. In addition, Zuber [8] has taken the Malkus theory of turbulent free convection as a basis for a general model of nucleate boiling.

In the analysis it was tactily assumed that film boiling was fully established. The following question may arise: since nucleate boiling is a much more efficient process than film boiling and if nature is attempting to maximize the heat transfer, why is the heat-transfer rate degraded at some critical temperature difference (burnout)? Apparently, these critical transition points must be looked upon as constraints. The variations of physical properties with temperature and pressure are thermodynamic constraints that always exist. Two examples illustrating how thermodynamic constraints determine what heat-transfer regime will exist are as follows:

(a) Above a certain wall temperature a liquid may be unable to wet the wall, which makes nucleate boiling impossible.

(b) Above the thermodynamic critical pressure of a fluid, no boiling can occur.

If Zuber's theory [8] of boiling with the use of the Malkus model of turbulent free convection is correct, then a general statement on boiling processes can be formulated: Boiling heat transfer, both nucleate and film, is an attempt by nature to maximize the rate of heat transfer into the system, subject to the general constraints imposed by thermodynamics and hydrodynamics.

Which regime of boiling, nucleate or film, actually obtains in a given situation cannot be predicted from such a general postulate, but must be regarded as calculable from the general constraints which are operating. However, given a regime and a choice among many alternatives (turbulence, cell distribution, etc.), nature will home in on that state of the system which maximizes the energy transport.

On a philosophical basis, why is nature so obliging in maximizing the rate of heat transfer? The explanation lies in entropy considerations. In essence, everything said about maximizing the energy transfer can be recast in terms of an entropy production; that is, if the rate of heat transfer is being maximized, the rate of entropy production is also being maximized. If the hot wall is considered a heat reservoir at temperature T_w and the bulk fluid is considered a heat sink at temperature T_s, the rate of entropy production of the universe is then

$$\frac{dS}{dt} = \frac{Q}{T_s} - \frac{Q}{T_w}$$

where Q is the total rate of heat transfer from the wall. Clearly, for fixed T_w and T_s, the rate of entropy production of the universe will be a maximum if the heat transport is a maximum.

The entropy formulation is more basic and will probably comprehend more physical situations than the energy formulation. The generalization that suggests itself, then, is that in all chaotic-type situations the average behavior of the system is such that the entropy production is maximized. This generalization is the macroscopic analog of the Second Law of Thermodynamics. Clearly, the importance of this nonequilibrium thermodynamic postulate is obvious, and its general validity warrants further study and investigation.

CONCLUSIONS

A theoretical analysis of pool film boiling from a horizontal surface yielded the following:

6

1. A theoretical expression for the heat-transfer coefficient that agrees within 4 percent of an experimentally confirmed expression proposed by Berenson
2. An optimum average vapor dome (bubble) radius that agrees within 5 percent of experimentally observed breakoff diameters
3. An optimum cell diameter (wavelength) which is intermediate between the critical and the most dangerous wavelength predicted by one-dimensional small-amplitude hydrodynamic stability theory

In addition, a tentative nonequilibrium thermodynamic hypothesis was proposed which states that nature maximizes the rate of entropy production in all systems characterized by macroscopic chaos, such as turbulence, nucleate boiling, and film boiling.

NOMENCLATURE

A_0, A_1, A_2, A_3	arbitrary constants
A_{ann}	areas of annular cell region, sq cm
A_{cell}	area of cell, sq cm
$A_\triangle$	area of curved triangular patch, sq cm
c_1, c_2	arbitrary constants
c_p	vapor specific heat at constant pressure, cal/(g)(°K)
D_B	breakoff diameter of vapor dome, cm
D_0	diameter of cell, cm
D_1	diameter of vapor dome, cm
E^2	differential operator defined by Eq. (15)
f	ratio of vapor dome to cell radius, R_1/R_0
g	local value of gravity, cm/sec²
g_c	conversion factor in Newton's law of motion, 1.0 g-cm/(sec²)(dyne)
h_{ann}	heat-transfer coefficient based on annular area, cal/(sq cm)(sec)(°K)
h_{cell}	heat-transfer coefficient based on cell area, cal/(sq cm)(sec)(°K)
h_{tot}	heat-transfer coefficient based on total plate area, cal/(sq cm)(sec)(°K)
k	vapor thermal conductivity, cal/(cm)(sec)(°K)
l	characteristic length given by Eq. (36), cm
P	absolute pressure, dyne/sq cm
P_s	pressure of system, dyne/sq cm
Q	rate of heat transfer, cal/sec
Q_{tot}	total rate of heat transfer from plate, cal/sec
R_0	radius of cell, cm
R_1	radius of vapor dome, cm
r	radial coordinate, cm
S	entropy, cal/°K
T	temperature, °K
ΔT	$T_w - T_s$, °K
T_s	saturation temperature, °K
T_w	wall temperature, °K
u	radial velocity, cm/sec
w	axial velocity, cm/sec
w_δ	evaporation velocity, cm/sec
x	dummy variable, cm
y	dummy variable, cm
z	axial coordinate, cm
α	thermal diffusivity, $k/\rho c_p$, cm/sec²
β^2	constant given by Eq. (41), 1/(sq cm)(sec)
δ	thickness of vapor film, cm
λ	latent heat of vaporization, cal/g
λ^*	modified latent heat of vaporization, cal/g
μ	vapor viscosity, g/(cm)(sec)
ρ	vapor density, g/cu cm
ρ_l	liquid density, g/cu cm
σ	surface tension, dyne/cm
ψ	stream function, cu cm/sec

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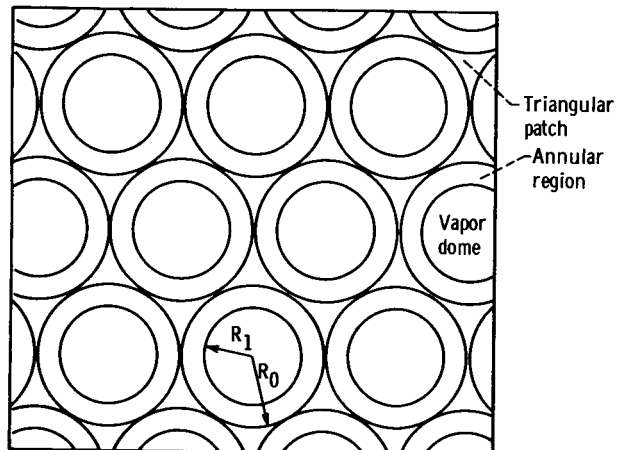
(a) Top view: Showing symmetrical distribution of cells on surface.
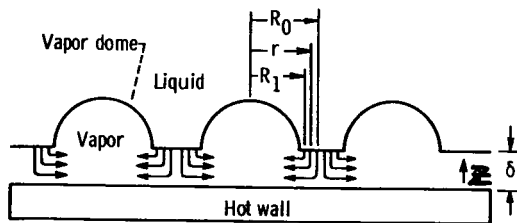
(b) Side view: Showing direction of vapor flow into domes, axial coordinate z, radial coordinate r, cell radius R_0, dome radius R_1, and gap thickness δ.
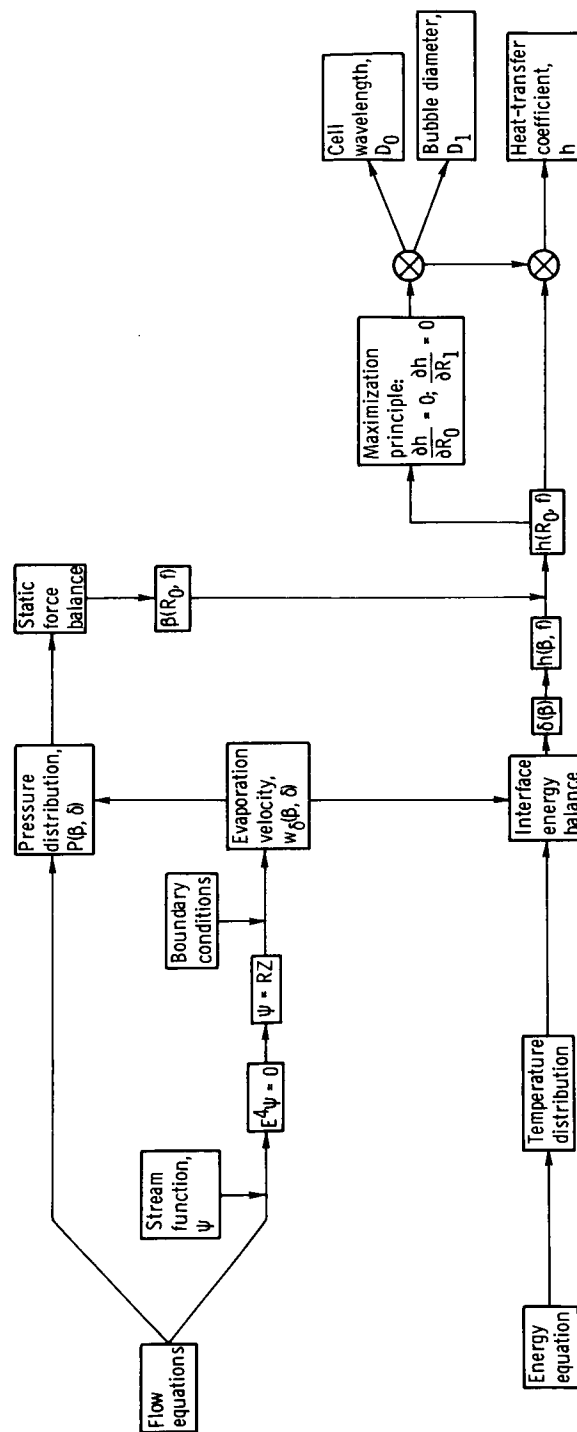
Figure 1. - Idealized cellular model for film boiling on horizontal surface.

Figure 2. – Solution flow chart.